MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

64th Annual

AMC 12 A

American Mathematics Contest 12 A

Tuesday, February 5, 2013

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 120 or above or finish in the top 2.5% on this AMC 10 will be invited to take the 31st annual American Invitational Mathematics Examination (AIME) on Thursday, March 14, 2013 or Wednesday, April 3, 2013. More details about the AIME and other information are on the back page of this test booklet.

2013

AMC 12 A

DO NOT OPEN UNTIL TUESDAY, FEBRUARY 5, 2013

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 5, 2013. Nothing is needed from inside this package until February 5.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

The American Mathematics Competitions

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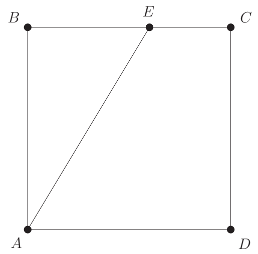
1. Square $ABCD$ has side length 10. Point E is on $\overline{BC}$, and the area of $\triangle ABE$ is 40. What is BE?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8



2. A softball team played ten games, scoring 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 runs. They lost by one run in exactly five games. In each of their other games, they scored twice as many runs as their opponent. How many total runs did their opponents score?

   (A) 35 **(B)** 40 **(C)** 45 **(D)** 50 **(E)** 55

3. A flower bouquet contains pink roses, red roses, pink carnations, and red carnations. One third of the pink flowers are roses, three fourths of the red flowers are carnations, and six tenths of the flowers are pink. What percent of the flowers are carnations?

   (A) 15 **(B)** 30 **(C)** 40 **(D)** 60 **(E)** 70

4. What is the value of
$$\frac{2^{2014}+2^{2012}}{2^{2014}-2^{2012}}?$$

   (A) -1 **(B)** 1 **(C)** $\frac{5}{3}$ **(D)** 2013 **(E)** 2^{4024}

5. Tom, Dorothy, and Sammy went on a vacation and agreed to split the costs evenly. During their trip Tom paid \$105, Dorothy paid \$125, and Sammy paid \$175. In order to share the costs equally, Tom gave Sammy t dollars, and Dorothy gave Sammy d dollars. What is $t-d$?

   (A) 15 **(B)** 20 **(C)** 25 **(D)** 30 **(E)** 35

6. In a recent basketball game, Shenille attempted only three-point shots and two-point shots. She was successful on 20% of her three-point shots and 30% of her two-point shots. Shenille attempted 30 shots. How many points did she score?

   (A) 12 **(B)** 18 **(C)** 24 **(D)** 30 **(E)** 36

7. The sequence $S_1, S_2, S_3, \ldots, S_{10}$ has the property that every term beginning with the third is the sum of the previous two. That is,

$$S_n = S_{n-2} + S_{n-1} \text{ for } n \geq 3.$$

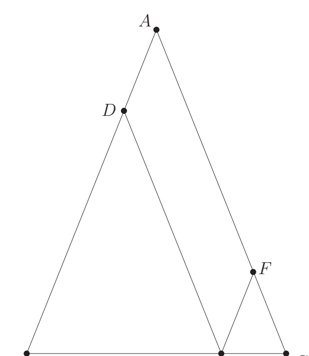
   Suppose that $S_9 = 110$ and $S_7 = 42$. What is S_4?

   (A) 4 **(B)** 6 **(C)** 10 **(D)** 12 **(E)** 16

8. Given that x and y are distinct nonzero real numbers such that $x + \frac{2}{x} = y + \frac{2}{y}$, what is xy?

   (A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** 4

9. In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points D, E, and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram $ADEF$?

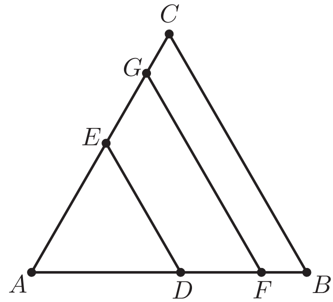


   (A) 48 **(B)** 52 **(C)** 56 **(D)** 60 **(E)** 72

10. Let S be the set of positive integers n for which $\frac{1}{n}$ has the repeating decimal representation $0.\overline{ab} = 0.ababab\ldots$, with a and b different digits. What is the sum of the elements of S?

   (A) 11 **(B)** 44 **(C)** 110 **(D)** 143 **(E)** 155

11. Triangle ABC is equilateral with $AB = 1$. Points E and G are on $\overline{AC}$ and points D and F are on $\overline{AB}$ such that both $\overline{DE}$ and $\overline{FG}$ are parallel to $\overline{BC}$. Furthermore, triangle ADE and trapezoids $DFGE$ and $FBCG$ all have the same perimeter. What is $DE + FG$?



(A) 1 **(B)** $\frac{3}{2}$ **(C)** $\frac{21}{13}$ **(D)** $\frac{13}{8}$ **(E)** $\frac{5}{3}$

12. The angles in a particular triangle are in arithmetic progression, and the side lengths are 4, 5, and x. The sum of the possible values of x equals $a + \sqrt{b} + \sqrt{c}$, where a, b, and c are positive integers. What is $a + b + c$?

(A) 36 **(B)** 38 **(C)** 40 **(D)** 42 **(E)** 44

13. Let points $A = (0, 0)$, $B = (1, 2)$, $C = (3, 3)$, and $D = (4, 0)$. Quadrilateral $ABCD$ is cut into equal area pieces by a line passing through A. This line intersects $\overline{CD}$ at point $\left(\frac{p}{q}, \frac{r}{s}\right)$, where these fractions are in lowest terms. What is $p + q + r + s$?

(A) 54 **(B)** 58 **(C)** 62 **(D)** 70 **(E)** 75

14. The sequence

$$\log_{12} 162, \log_{12} x, \log_{12} y, \log_{12} z, \log_{12} 1250$$

is an arithmetic progression. What is x?

(A) $125\sqrt{3}$ **(B)** 270 **(C)** $162\sqrt{5}$ **(D)** 434 **(E)** $225\sqrt{6}$

15. Rabbits Peter and Pauline have three offspring—Flopsie, Mopsie, and Cottontail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

(A) 96 **(B)** 108 **(C)** 156 **(D)** 204 **(E)** 372

16. A, B, and C are three piles of rocks. The mean weight of the rocks in A is 40 pounds, the mean weight of the rocks in B is 50 pounds, the mean weight of the rocks in the combined piles A and B is 43 pounds, and the mean weight of the rocks in the combined piles A and C is 44 pounds. What is the greatest possible integer value for the mean in pounds of the rocks in the combined piles B and C?

    (A) 55 **(B)** 56 **(C)** 57 **(D)** 58 **(E)** 59

17. A group of 12 pirates agree to divide a treasure chest of gold coins among themselves as follows. The k^{th} pirate to take a share takes $\frac{k}{12}$ of the coins that remain in the chest. The number of coins initially in the chest is the smallest number for which this arrangement will allow each pirate to receive a positive whole number of coins. How many coins does the 12^{th} pirate receive?

    (A) 720 **(B)** 1296 **(C)** 1728 **(D)** 1925 **(E)** 3850

18. Six spheres of radius 1 are positioned so that their centers are at the vertices of a regular hexagon of side length 2. The six spheres are internally tangent to a larger sphere whose center is the center of the hexagon. An eighth sphere is externally tangent to the six smaller spheres and internally tangent to the larger sphere. What is the radius of this eighth sphere?

    (A) $\sqrt{2}$ **(B)** $\dfrac{3}{2}$ **(C)** $\dfrac{5}{3}$ **(D)** $\sqrt{3}$ **(E)** 2

19. In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

    (A) 11 **(B)** 28 **(C)** 33 **(D)** 61 **(E)** 72

20. Let S be the set $\{1, 2, 3, \ldots, 19\}$. For $a, b \in S$, define $a \succ b$ to mean that either $0 < a - b \leq 9$ or $b - a > 9$. How many ordered triples (x, y, z) of elements of S have the property that $x \succ y$, $y \succ z$, and $z \succ x$?

    (A) 810 **(B)** 855 **(C)** 900 **(D)** 950 **(E)** 988

21. Consider

$$A = \log(2013 + \log(2012 + \log(2011 + \log(\cdots + \log(3 + \log 2)\cdots)))).$$

Which of the following intervals contains A?

(A) $(\log 2016, \log 2017)$

(B) $(\log 2017, \log 2018)$

(C) $(\log 2018, \log 2019)$

(D) $(\log 2019, \log 2020)$

(E) $(\log 2020, \log 2021)$

22. A palindrome is a nonnegative integer number that reads the same forwards and backwards when written in base 10 with no leading zeros. A 6-digit palindrome n is chosen uniformly at random. What is the probability that $\frac{n}{11}$ is also a palindrome?

(A) $\dfrac{8}{25}$ **(B)** $\dfrac{33}{100}$ **(C)** $\dfrac{7}{20}$ **(D)** $\dfrac{9}{25}$ **(E)** $\dfrac{11}{30}$

23. $ABCD$ is a square of side length $\sqrt{3}+1$. Point P is on $\overline{AC}$ such that $AP = \sqrt{2}$. The square region bounded by $ABCD$ is rotated 90° counterclockwise with center P, sweeping out a region whose area is $\frac{1}{c}(a\pi + b)$, where a, b, and c are positive integers and $\gcd(a, b, c) = 1$. What is $a + b + c$?

(A) 15 **(B)** 17 **(C)** 19 **(D)** 21 **(E)** 23

24. Three distinct segments are chosen at random among the segments whose end-points are the vertices of a regular 12-gon. What is the probability that the lengths of these three segments are the three side lengths of a triangle with positive area?

(A) $\dfrac{553}{715}$ **(B)** $\dfrac{443}{572}$ **(C)** $\dfrac{111}{143}$ **(D)** $\dfrac{81}{104}$ **(E)** $\dfrac{223}{286}$

25. Let $f : \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = z^2 + iz + 1$. How many complex numbers z are there such that $\text{Im}(z) > 0$ and both the real and the imaginary parts of $f(z)$ are integers with absolute value at most 10?

(A) 399 **(B)** 401 **(C)** 413 **(D)** 431 **(E)** 441

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WRITE TO US!

Correspondence about the problems and solutions for this AMC 12
and orders for publications should be addressed to:

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The problems and solutions for this AMC 12 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:

Prof. Bernardo M. Abrego

2013 AIME

The 31st annual AIME will be held on Thursday, March 14, with the alternate on Wednesday, April 3. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 42nd Annual USA Mathematical Olympiad (USAMO) on April 30-May 1, 2013. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
amc.maa.org